

JANUARY 2010

**Paper 02 – General Proficiency**

*2 hours 40 minutes*

**05 JANUARY 2010 (a.m.)**

**INSTRUCTIONS TO CANDIDATES**

1. Answer ALL questions in Section I, and ANY TWO in Section II.
2. Write your answers in the booklet provided.
3. All working must be clearly shown.
4. A list of formulae is provided on page 2 of this booklet.

**Examination Materials**

Electronic calculator (non-programmable)

Geometry set

Mathematical tables (provided)

Graph paper (provided)

**LIST OF FORMULAE**

Volume of a prism  $V = Ah$  where  $A$  is the area of a cross-section and  $h$  is the perpendicular length.

Volume of cylinder  $V = \pi r^2 h$  where  $r$  is the radius of the base and  $h$  is the perpendicular height.

Volume of a right pyramid  $V = \frac{1}{3}Ah$  where  $A$  is the area of the base and  $h$  is the perpendicular height.

Circumference  $C = 2\pi r$  where  $r$  is the radius of the circle.

Area of a circle  $A = \pi r^2$  where  $r$  is the radius of the circle.

Area of trapezium  $A = \frac{1}{2}(a + b)h$  where  $a$  and  $b$  are the lengths of the parallel sides and  $h$  is the perpendicular distance between the parallel sides.

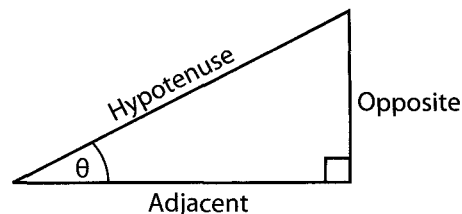
Roots of quadratic equations If  $ax^2 + bx + c = 0$ ,  
then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of triangle

Area of  $\Delta = \frac{1}{2}bh$  where  $b$  is the length of the base and  $h$  is the perpendicular height

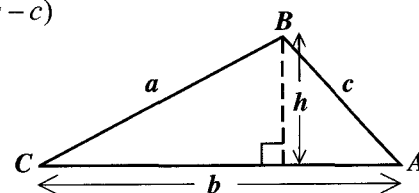
$$\text{Area of } \Delta ABC = \frac{1}{2}ab \sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

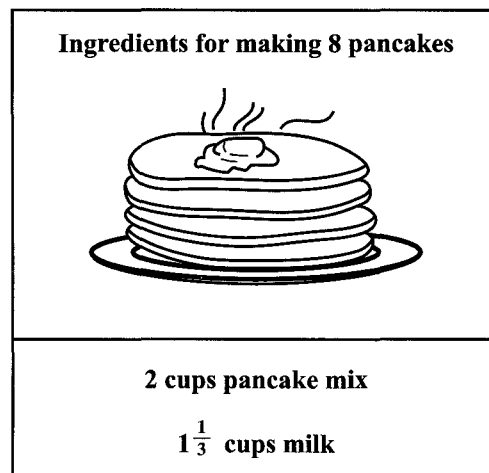
1. (a) Using a calculator, or otherwise, calculate the exact value of

$$\frac{2.76}{0.8} + 8.7^2 \quad (3 \text{ marks})$$

- (b) In a certain company, a salesman is paid a fixed salary of \$3 140 per month plus an annual commission of 2% on the TOTAL value of cars sold for the year. If the salesman sold cars valued at \$720 000 in 2009, calculate

- (i) his fixed salary for the year (1 mark)
- (ii) the amount he received in commission for the year (1 mark)
- (iii) his TOTAL income for the year. (1 mark)

- (c) The ingredients for making pancakes are shown in the diagram below.



- (i) Ryan wishes to make 12 pancakes using the instructions given above. Calculate the number of cups of pancake mix he must use. (2 marks)
- (ii) Neisha used 5 cups of milk to make pancakes using the same instructions. How many pancakes did she make? (3 marks)

**Total 11 marks**

2. (a) Given that  $a = 6$ ,  $b = -4$  and  $c = 8$ , calculate the value of

$$\frac{a^2 + b}{c - b} \quad (3 \text{ marks})$$

- (b) Simplify the expression:

(i)  $3(x - y) + 4(x + 2y)$  (2 marks)

(ii)  $\frac{4x^2 \times 3x^4}{6x^3}$  (3 marks)

- (c) (i) Solve the inequality

$$x - 3 < 3x - 7 \quad (3 \text{ marks})$$

- (ii) If  $x$  is an integer, determine the SMALLEST value of  $x$  that satisfies the inequality in (c) (i) above. (1 mark)

**Total 12 marks**

3. (a)  $T$  and  $E$  are subsets of a universal set,  $U$ , such that:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$T = \{\text{multiples of } 3\}$$

$$E = \{\text{even numbers}\}$$

- (i) Draw a Venn diagram to represent this information. (4 marks)

- (ii) List the members of the set

a)  $T \cap E$  (1 mark)

b)  $(T \cup E)'$  (1 mark)

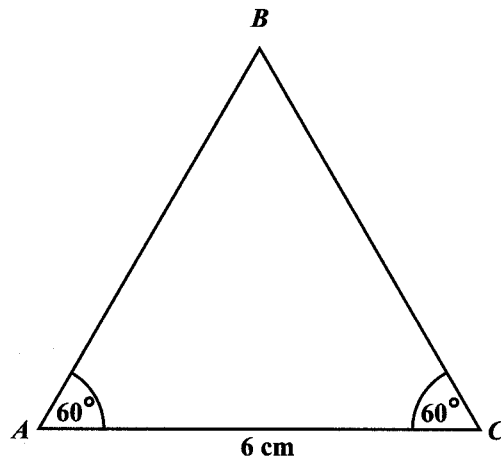
(b) Using a pencil, a ruler, and a pair of compasses only:

(i) Construct, accurately, the triangle  $ABC$  shown below, where,

$$AC = 6 \text{ cm}$$

$$\angle ACB = 60^\circ$$

$$\angle CAB = 60^\circ$$



( 3 marks)

(ii) Complete the diagram to show the kite,  $ABCD$ , in which  $AD = 5 \text{ cm}$ .

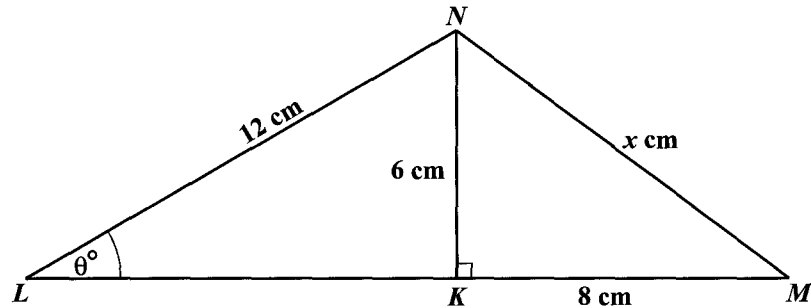
( 2 marks)

(iii) Measure and state the size of  $\angle DAC$ .

( 1 mark )

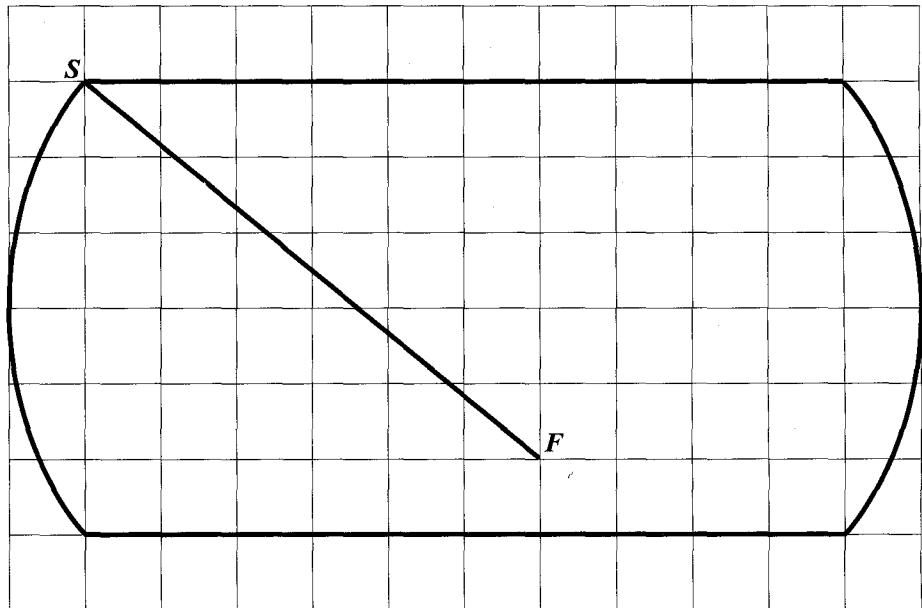
**Total 12 marks**

4. (a) The diagram below, **not drawn to scale**, shows a triangle  $LMN$  with  $LN = 12$  cm,  $NM = x$  cm and  $\angle NLM = \theta^\circ$ . The point  $K$  on  $LM$  is such that  $NK$  is perpendicular to  $LM$ ,  $NK = 6$  cm, and  $KM = 8$  cm.



Calculate the value of

- (i)  $x$  ( 2 marks)
- (ii)  $\theta$ . ( 3 marks)
- (b) The diagram below shows a map of a playing field drawn on a grid of 1 cm squares. **The scale of the map is 1 : 1 250.**



- (i) Measure and state, in centimetres, the distance from  $S$  to  $F$  on the map. ( 1 mark )
- (ii) Calculate the distance, in metres, from  $S$  to  $F$  on the ACTUAL playing field. ( 2 marks)

- (iii) Daniel ran the distance from  $S$  to  $F$  in 9.72 seconds. Calculate his average speed in
- a) m/s
- b) km/h
- giving your answer correct to 3 significant figures. ( 3 marks)

**Total 11 marks**

5. (a) A straight line passes through the point  $T(4,1)$  and has a gradient of  $\frac{3}{5}$ . Determine the equation of this line. ( 3 marks)
- (b) (i) **Using a scale of 1 cm to represent 1 unit on both axes**, draw the triangle  $ABC$  with vertices  $A(2,3)$ ,  $B(5,3)$  and  $C(3,6)$ . ( 3 marks)
- (ii) On the same axes used in (b) (i), draw and label the line  $y = 2$ . ( 1 mark)
- (iii) Draw the image of triangle  $ABC$  under a reflection in the line  $y = 2$ . Label the image  $A'B'C'$ . ( 2 marks)
- (iv) Draw a new triangle  $A''B''C''$  with vertices  $A''(-7,4)$ ,  $B''(-4,4)$  and  $C''(-6,7)$ . ( 1 mark)
- (v) Name and describe the single transformation that maps triangle  $ABC$  onto triangle  $A''B''C''$ . ( 2 marks)

**Total 12 marks**

6. A class of 26 students each recorded the distance travelled to school. The distance, to the nearest km, is recorded below:

21    11    3    22    6    32    22    18    28  
 26    16    17    34    12    25    8    19    14  
 39    17    22    24    30    18    13    23

- (a) Copy and complete the frequency table to represent this data.

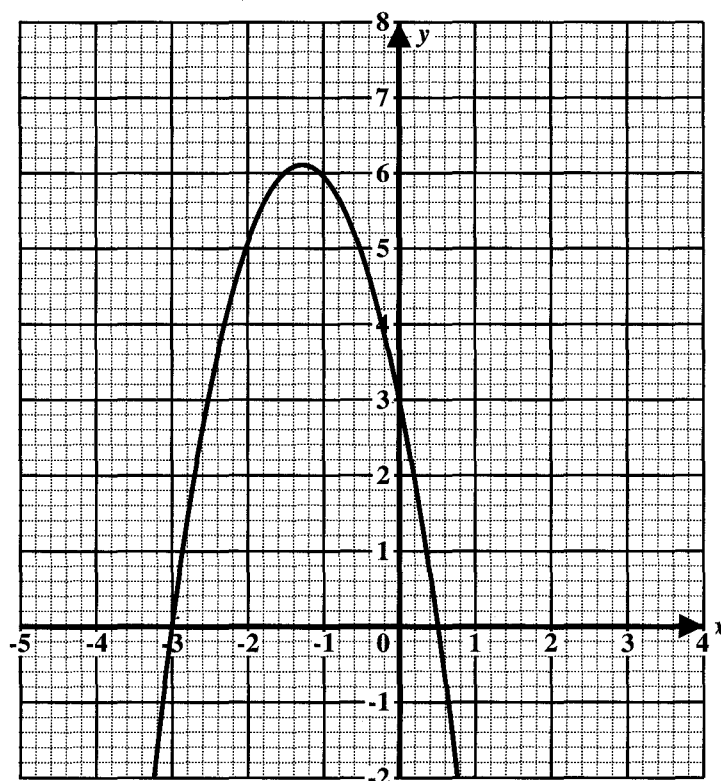
Distance in kilometres	Frequency
1 - 5	1
6 - 10	2
11 - 15	4
16 - 20	6
21 - 25	
26 - 30	
31 - 35	
36 - 40	

( 2 marks)

- (b) Using a scale of 2 cm to represent 5 km on the horizontal axis and a scale of 1 cm to represent 1 student on the vertical axis, draw a histogram to represent the data.  
( 5 marks)
- (c) Calculate the probability that a student chosen at random from this class recorded the distance travelled to school as 26 km or more.  
( 2 marks)
- (d) The P.T.A. plans to set up a transportation service for the school. Which average, mean, mode or median, is MOST appropriate for estimating the cost of the service? Give a reason for your answer.  
( 2 marks)

**Total 11 marks**

7. The graph shown below represents a function of the form:  $f(x) = ax^2 + bx + c$ .

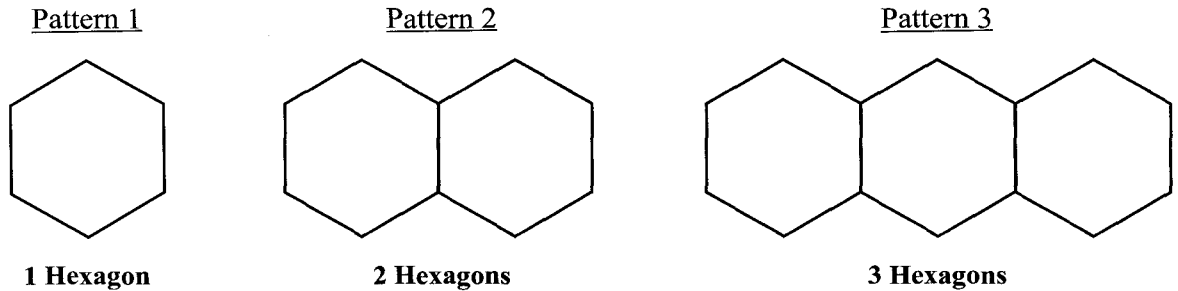


Using the graph above, determine

- (i) the value of  $f(x)$  when  $x = 0$  ( 1 mark )
- (ii) the values of  $x$  when  $f(x) = 0$  ( 2 marks )
- (iii) the coordinates of the maximum point ( 2 marks )
- (iv) the equation of the axis of symmetry ( 2 marks )
- (v) the values of  $x$  when  $f(x) = 5$  ( 2 marks )
- (vi) the interval within which  $x$  lies when  $f(x) > 5$ . Write your answer in the form  $a < x < b$ . ( 2 marks )

**Total 11 marks**

8. Bianca makes hexagons using sticks of equal length. She then creates patterns by joining the hexagons together. Patterns 1, 2 and 3 are shown below:



The table below shows the number of hexagons in EACH pattern created and the number of sticks used to make EACH pattern.

<b>Number of hexagons in the pattern</b>	1	2	3	4	5	20	$n$
<b>Number of sticks used for the pattern</b>	6	11	16	$x$	$y$	$z$	$S$

- (a) Determine the values of
- (i)  $x$  ( 2 marks)
- (ii)  $y$  ( 2 marks)
- (iii)  $z$ . ( 2 marks)
- (b) Write down an expression for  $S$  in terms of  $n$ , where  $S$  represents the number of sticks used to make a pattern of  $n$  hexagons. ( 2 marks)
- (c) Bianca used a total of 76 sticks to make a pattern of  $h$  hexagons. Determine the value of  $h$ . ( 2 marks)

**Total 10 marks**

## SECTION II

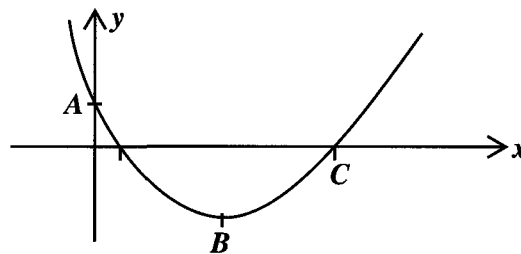
Answer TWO questions in this section.

## RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The relationship between kinetic energy,  $E$ , mass,  $m$ , and velocity,  $v$ , for a moving particle is

$$E = \frac{1}{2}mv^2.$$

- (i) Express  $v$  in terms of  $E$  and  $m$ . ( 3 marks)
- (ii) Determine the value of  $v$  when  $E = 45$  and  $m = 13$ . ( 2 marks)
- (b) Given  $g(x) = 3x^2 - 8x + 2$ ,
- (i) write  $g(x)$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c \in \mathbf{R}$  ( 3 marks)
- (ii) solve the equation  $g(x) = 0$ , writing your answer(s) correct to 2 decimal places. ( 4 marks)
- (iii) A sketch of the graph of  $g(x)$  is shown below.



Copy the sketch and state

- a) the  $y$ -coordinate of  $A$
- b) the  $x$ -coordinate of  $C$
- c) the  $x$  and  $y$ -coordinates of  $B$ . ( 3 marks)

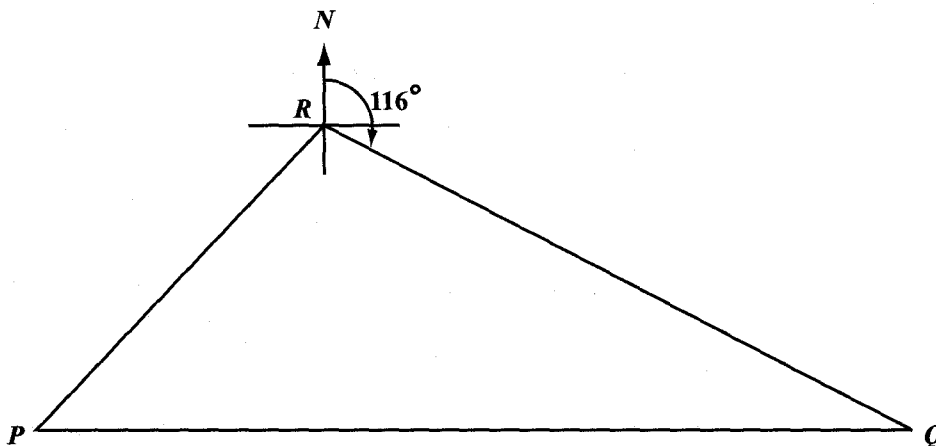
**Total 15 marks**

10. (a) The manager of a pizza shop wishes to make  $x$  small pizzas and  $y$  large pizzas. His oven holds no more than 20 pizzas.
- (i) Write an inequality to represent the given condition. ( 2 marks)
- The ingredients for each small pizza cost \$15 and for each large pizza \$30. The manager plans to spend no more than \$450 on ingredients.
- (ii) Write an inequality to represent this condition. ( 2 marks)
- (b) (i) **Using a scale of 2 cm on the  $x$ -axis to represent 5 small pizzas and 2 cm on the  $y$ -axis to represent 5 large pizzas**, draw the graphs of the lines associated with the inequalities at (a) (i) and (a) (ii) above. ( 4 marks)
- (ii) Shade the region which is defined by ALL of the following combined:
- the inequalities written at (a) (i) and (a) (ii)
  - the inequalities  $x \geq 0$  and  $y \geq 0$
- ( 1 mark )
- (iii) Using your graph, state the coordinates of the vertices of the shaded region. ( 2 marks)
- (c) The pizza shop makes a profit of \$8 on the sale of EACH small pizza and \$20 on the sale of EACH large pizza. All the pizzas that were made were sold.
- (i) Write an expression in  $x$  and  $y$  for the TOTAL profit made on the sale of the pizzas. ( 1 mark )
- (ii) Use the coordinates of the vertices given at (b) (iii) to determine the MAXIMUM profit. ( 3 marks)

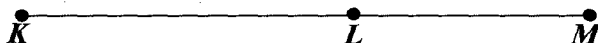
**Total 15 marks**

### GEOMETRY AND TRIGONOMETRY

11. (a) The diagram below, **not drawn to scale**, shows three stations  $P$ ,  $Q$  and  $R$ , such that the bearing of  $Q$  from  $R$  is  $116^\circ$  and the bearing of  $P$  from  $R$  is  $242^\circ$ . The vertical line at  $R$  shows the North direction.



- (i) Show that angle  $PRQ = 126^\circ$ . ( 2 marks)
- (ii) Given that  $PR = 38$  metres and  $QR = 102$  metres, calculate the distance  $PQ$ , giving your answer to the nearest metre. ( 3 marks)
- (b)  $K$ ,  $L$  and  $M$  are points along a straight line on a horizontal plane, as shown below.



A vertical pole,  $SK$ , is positioned such that the angles of elevation of the top of the pole  $S$  from  $L$  and  $M$  are  $21^\circ$  and  $14^\circ$  respectively.

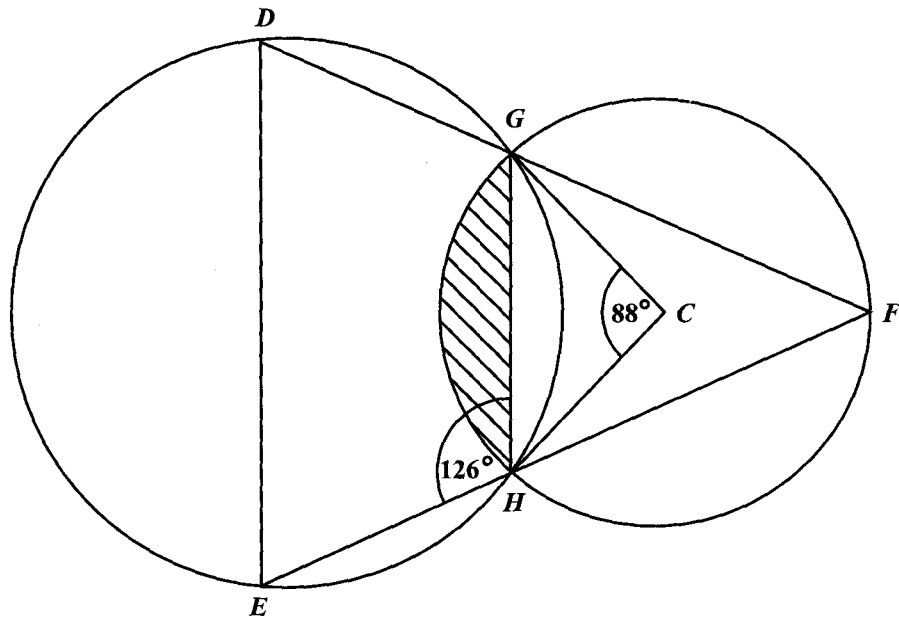
The height of the pole,  $SK$ , is 10 metres.

- (i) Copy and complete the diagram to show the pole  $SK$  and the angles of elevation of  $S$  from  $L$  and  $M$ . ( 4 marks)
- (ii) Calculate, correct to ONE decimal place,
- a) the length of  $KL$
  - b) the length of  $LM$ . ( 6 marks)

**Total 15 marks**

12. (a) The diagram below, **not drawn to scale**, shows two circles.  $C$  is the centre of the smaller circle,  $GH$  is a common chord and  $DEF$  is a triangle.

Angle  $GCH = 88^\circ$  and angle  $GHE = 126^\circ$ .



Calculate, **giving reasons for your answer**, the measure of angle

- (i)  $GFH$  ( 2 marks)
- (ii)  $GDE$  ( 3 marks)
- (iii)  $DEF$ . ( 2 marks)

- (b) Use  $\pi = 3.14$  in this part of the question.

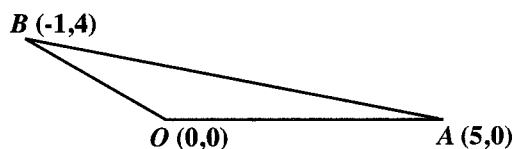
Given that  $GC = 4$  cm, calculate the area of

- (i) triangle  $GCH$  ( 3 marks)
- (ii) the minor sector bounded by arc  $GH$  and radii  $GC$  and  $HC$  ( 3 marks)
- (iii) the shaded segment. ( 2 marks)

**Total 15 marks**

## VECTORS AND MATRICES

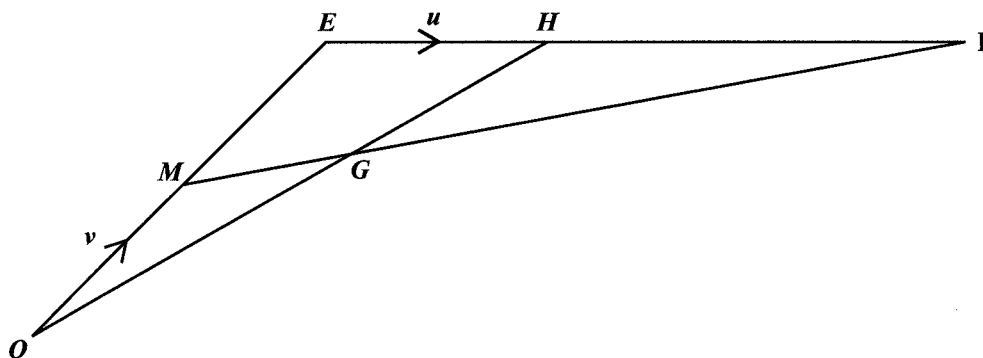
13. (a) The figure below, **not drawn to scale**, shows the points  $O(0,0)$ ,  $A(5,0)$  and  $B(-1,4)$  which are the vertices of a triangle  $OAB$ .



- (i) Express in the form  $\begin{bmatrix} a \\ b \end{bmatrix}$  the vectors
- a)  $\vec{OB}$
- b)  $\vec{OA} + \vec{OB}$  ( 3 marks)
- (ii) If  $M(x,y)$  is the midpoint of  $AB$ , determine the values of  $x$  and  $y$ . ( 2 marks)

- (b) In the figure below, **not drawn to scale**,  $OE$ ,  $EF$  and  $MF$  are straight lines. The point  $H$  is such that  $EF = 3EH$ . The point  $G$  is such that  $MF = 5MG$ .  $M$  is the midpoint of  $OE$ .

The vector  $\vec{OM} = v$  and  $\vec{EH} = u$ .



- (i) Write in terms of  $u$  and/or  $v$ , an expression for:
- a)  $\vec{HF}$  ( 1 mark)
- b)  $\vec{MF}$  ( 2 marks)
- c)  $\vec{OH}$  ( 2 marks)
- (ii) Show that  $\vec{OG} = \frac{3}{5} (2v + u)$  ( 2 marks)
- (iii) Hence, prove that  $O$ ,  $G$  and  $H$  lie on a straight line. ( 3 marks)

**Total 15 marks**

14. (a)  $L$  and  $N$  are two matrices where

$$L = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}.$$

Evaluate  $L - N^2$ . ( 3 marks)

- (b) The matrix,  $M$ , is given as  $M = \begin{bmatrix} x & 12 \\ 3 & x \end{bmatrix}$ . Calculate the values of  $x$  for which  $M$  is singular. ( 2 marks)

- (c) A geometric transformation,  $R$ , maps the point  $(2,1)$  onto  $(-1,2)$ .

Given that  $R = \begin{bmatrix} 0 & p \\ q & 0 \end{bmatrix}$ , calculate the values of  $p$  and  $q$ . ( 3 marks)

- (d) A translation,  $T = \begin{bmatrix} r \\ s \end{bmatrix}$  maps the point  $(5,3)$  onto  $(1,1)$ . Determine the values of  $r$  and  $s$ .

( 3 marks)

- (e) Determine the coordinates of the image of  $(8,5)$  under the combined transformation,  $R$  followed by  $T$ . ( 4 marks)

**Total 15 marks**

**END OF TEST**